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LORENTZIAN PARA-SASAKIAN MANIFOLD

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Abstract: In this paper work is done on some properties of the contact CR-submanifolds. Lorentzian para-Sasakian manifolds are D-totally geodesic and $D^{\perp}$-totally geodesic.

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Key words : Contact CR-submanifold, Lorentzian para-Sasakian manifold, D-totally geodesic, $D^{\perp}$-totally geodesic,

1 Introduction

In 1978, Bejancu introduced the notion of C.R. Submanifold of a Kaehler manifold[1],Since then several works are going on C.R. Submanifolds. Matsumoto[2] introduced the idea of Lorentzian para- contact structure and studied its several properties. Later Mihai and Matsumoto [3] introduced the same notion independently and they obtained several results on this manifolds. Lorentzian para-Sasakian manifold have also been studied by U.C.De,A.A.Shaikh and Mihai[4] and Venkatesh, C.S.Bagewadi and Pradeep Kumar K.T.[5]. Recently Ahmet yildiz, U.C.De and Erhan Ata[6] worked on Lorentzian para-Sasakian manifolds and introduced the new concept called generalized η-Einstien manifold in a Lorentzian para-Sasakian manifolds.
In this paper In this paper work is done on some properties of the contact CR-submanifolds. Lorentzian para-Sasakian manifolds are D-totally geodesic and $D^{\perp}$-totally geodesic.

2. Preliminaries

Let M be an (2n+1)-dimensional almost contact metric manifold with indefinite almost contact metric structure (φ, ξ, η, g) then they satisfies

2.1. $$\varphi^2 = -I + \eta \otimes \xi,$$

$$\eta(\xi) = 1, \quad \varphi\xi = 0, \eta o \varphi = 0,$$

2.2. $$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\,\eta(Y),$$

$$g(\varphi X, Y) = -g(X, \varphi Y),$$

2.3. $$g(X, \xi) = \eta(X),$$

For all vector fields X,Y on M.

An almost metric structure (φ, ξ, η, g) is called an Lorentzian para-Sasakian manifold if

2.4 $$(\tilde{\nabla}_X \varphi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

Where $\tilde{\nabla}$ is the Levi-Civita (L-C) connection for a semi-Riemannian metric g. Also we have

2.5 $$\tilde{\nabla}_X \xi = \varphi X,$$

Where $X \epsilon TM$.

The Gauss and Weingarten formulae are as follows

2.6 $$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y),$$

2.7 $$\tilde{\nabla}_X N = -A_N X + \nabla_X^{\perp} N,$$

for any $X, Y \in TM$ and $N \in T^{\perp}M$, where $\nabla^{\perp}$ is the connection on the normal bundle $T^{\perp}M$, h is the second fundamental form and A_N is the Weingarten map associated with N via

2.8 $$g(A_N X, Y) = g(h(X, Y), N).$$

The equation of Gauss is given by

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2.9 $\tilde{R}(X, Y, Z, W) = R(X, Y, Z, W) + g(h(X, Z), h(Y, W)) - g(h(X, W), h(Y, Z))$,

where $\tilde{R}$ (resp. R) is the curvature tensor of $\tilde{M}$ (resp. M).

For any $x \in M$, $X_x \in T_xM$ and $N \in T^{\perp}M$, we write,

2.10 $$X = PX + QX,$$

2.11 $$\varphi N = BN + CN,$$

where PX (resp. BN) denotes the tangential part of X (resp. φN) and QX (resp. CN) denotes the normal part of X (resp. φN) respectively.

Using (2.6), (2.7), (2.10), (2.11) in (2.4) after a brief calculation we obtain on com paring the horizontal, vertical and normal parts,

2.12. $P\nabla_X \varphi PY - PA_{\varphi QY}X = \varphi P\nabla_X Y + g(PX, Y)\xi + \eta(Y)PX + 2\eta(Y)\eta(X)$,

2.13 $Q\nabla_X \varphi PY + QA_{\varphi QY}X = Bh(X, Y) + g(QX, Y)\xi + \eta(Y)QX$,

2.14 $h(X, \varphi PY) + \nabla^{\perp}_X \varphi QY = \varphi Q\nabla_X Y + Ch(X, Y)$.

From (2.5) we have

2.15 $\nabla_X \xi = \varphi PX$,

2.16 $h(X, \xi) = \varphi QX$,

Also we have

2.17 $h(X, \xi) = 0$ if $X \in D$,

2.18 $\nabla_X \xi = 0$,

2.19 $h(\xi, \xi) = 0$,

2.20 $A_N \xi \in D^{\perp}$.

3. D-totally geodesic and $D^{\perp}$-totally geodesic contact CR-Submanifolds of Lorentzian para-Sasakian manifold

Definition: A contact CR-submanifold M of an Lorentzian para-Sasakian manifold $\tilde{M}$ is called D-totally geodesic (resp. $D^{\perp}$-totally geodesic) if $h(X, Y)= 0$, $\forall$ $X, Y \in D$ (resp. $X, Y \in D^{\perp}$).

Proposition 1 Let M be a contact CR-submanifold of an Lorentzian para-Sasakian manifold. Then M is a D-totally geodesic if and only if $A_N X \in D^{\perp}$ for each $X \in D$ and N a normal vector field to M.

Proof : Let M be D-totally geodesic. Then from (2.8) we get

$$g(h(X, Y), N)= g(A_N X, Y)= 0.$$

So if

$$h(X, Y) = 0, \forall\ X, Y \in D$$
$$\text{i.e.,} \qquad A_N X \in D^{\perp}.$$

Conversely, let $A_N X \in D^{\perp}$. Then for $X, Y \in D$ we can obtain,

$$g(A_N X, Y)= 0 = g(h(X, Y), N),$$
$$\text{i.e.,} \qquad h(X, Y) = 0$$

$\forall$ $X, Y \in D$, which implies that M is D-totally geodesic. Thus our proof is complete.

Proposition 2 : Let M be a contact CR-submanifold of Lorentzian para-Sasakian manifold $\tilde{M}$. Then M is $D^{\perp}$-totally geodesic if and only if $A_N X \in D$ for each $X \in D^{\perp}$ and N a normal vector field to M.

Proof: The proof follows immediately from the above proposition.

Concerning the integrability of the horizontal distribution D and vertical distribution $D^{\perp}$ on M, we can state the following theorem :

Theorem: Let M be a contact CR-submanifold of an Lorentzian para-Sasakian manifold. If M is ξ-horizontal, then the distribution D is integrable iff

3.1 $$h(X, \varphi Y) = h(\varphi X, Y),$$

$\forall$ X, Y $\in$ D. If M is ξ-vertical then the distribution $D^{\perp}$ is integrable iff

3.2 $$A_{\varphi X}Y - A_{\varphi Y}X = \eta(Y)X - \eta(X)Y, \qquad \forall\ X, Y \in D^{\perp}.$$

Proof: If M is ξ-horizontal, then using (2.14) we get,

$$h(X, \varphi PY) = \varphi Q \nabla_X Y + Ch(X, Y)$$

$\forall$ X, Y $\in$ D. Therefore $[X, Y] \in D$ iff $h(X, \varphi Y) = h(Y, \varphi X)$

Hence, if M is ξ-horizontal, $[X, Y] \in D$ iff $h(X, \varphi Y) = h(\varphi X, Y)$.

Again using (2.14) we get,

$$\nabla^{\perp}\varphi Y = Ch(X, Y) + \varphi Q \nabla_X Y \quad \text{for } X, Y \in D^{\perp}.$$

After some calculations we see that

3.3. $$\tilde{\nabla}_X \varphi Y = g(X, Y)\xi + \eta(Y)X + 2\eta(Y)\eta(X)\xi + \varphi P \nabla_X Y + \varphi Q \nabla_X Y + Bh(X, Y) + Ch(X, Y).$$

Again from (2.7) and (3.3) we get,

3.4 $$\nabla_X^{\perp}\varphi Y = A_{\varphi Y} X + g(X,Y)\xi + 2\eta(Y)\eta(X)\xi + \varphi P \nabla_X Y + \varphi Q \nabla_X Y + Bh(X, Y) + Ch(X, Y).$$

for X, Y $\in D^{\perp}$. From (3.4) and (3.3) we can write,

3.5 $$\varphi P \nabla_X Y = -A_{\varphi Y}X - g(X, Y)\xi - \eta(Y)X - 2\eta(Y)\eta(X)\xi - Bh(X, Y).$$

Interchanging X and Y in (3.5) we get,

3.6 $$\varphi P \nabla_Y X = -A_{\varphi X}Y - g(X, Y)\xi - \eta(X)Y - 2\eta(Y)\eta(X)\xi - Bh(X, Y).$$

Substracting (3.5) from (3.6) we have,

$$\varphi P[X, Y] = -A_{\varphi Y}X + A_{\varphi X}Y - \eta(Y)X + \eta(X)Y.$$

Now since M is ξ-vertical, $[X, Y] \in D^{\perp}$ iff,

$$A_{\varphi X}Y - A_{\varphi Y}X = \eta(Y)X - \eta(X)Y.$$

So the proof is complete.

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